Long Range Interactions Between Neutral Atoms

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Abstract. In a magneto-otpical trap (MOT) where atoms can be cooled and trapped using quasi-resonant laser light, the effect of rescattered light limits the spatial density of the atoms. Here we draw an analogy between the forces associated to this multiple scattering and a Coulomb type long range interaction. A MOT in the density limited regime can thus be interpreted as a non neutral plasma with weakly charged particles. For very large samples, non linear terms in the cooling and trapping forces can lead to self sustained instabilities via a supercritical Hopf bifurcation.

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INTRODUCTION

The effect of multiple scattering on the dynamics of the atoms is well known in the community of laser cooling of atoms, as multiple scattering has been a major limitation to obtain large phase space densities in cold atomic traps. Bose-Einstein condensation in dilute atomic vapors has only been achieved after switching off all laser fields and using evaporation techniques[1]. On the other side, multiple scattering of light in cold atoms has been used to study coherent light transport and wave localization in random media[2]. This has motivated an investigation of yet unexplored regimes of cold atomic clouds, namely the limit of very large numbers of atoms in the presence of quasi-resonant light. Here we do not focus on the properties of the scattered light but on the mechanical effects of this light on the atoms. We first present the mechanical effects of the scattered light in a regime where the size of the cloud of cold atoms increases with the number of trapped atoms, and explain how this can be related to Coulomb type long range interactions. A MOT in the density limited regime can thus be interpreted as a non neutral plasma with weakly charged particles.

We then focus on a new phenomenon, namely a collective instabilities which we have observed and identified as a supercritical Hopf bifurcation.

MOT : A DAMPED HARMONIC OSCILLATOR

A simple model of a MOT can be obtained by adding the radiation pressure forces acting on the atoms, as proposed in 1975 by T. Hänsch and A. Shawlow[3] for neutral atoms and by D. Wineland and H. Dehmelt[4] for ions. Consider the case of two lasers counterpropagating along Oz with the same frequency $\omega_L = \omega_{at} + \delta$. This can be generalized to three dimensions, but for simplicity we will restrict ourselves to one dimension. An atom in such a laser configuration is subject to light-induced forces, including near-resonant radiation pressure and the dipole force. However, a basic description, within the so-called Doppler cooling picture, can be given by simply adding independently the near-resonant radiation pressure forces of the two propagating laser fields. This approximation neglects dipole forces, i.e. the well-known Sisyphus effect. Nevertheless many qualitative features of large MOTs can be understood without such dipole forces.

Let us consider the case of the atoms in free space with no magnetic field. For low light intensities, the force for an atom moving with velocity v can be obtained by adding the two quasi-resonant radiation pressure forces :

$$f(v) = \hbar k \frac{\Gamma}{2} \frac{I_{inc}}{I_{sat}} \left(\frac{\Gamma^2/4}{(\delta - kv)^2 + \Gamma^2/4} + \frac{\Gamma^2/4}{(\delta + kv)^2 + \Gamma^2/4} \right) \qquad (1)$$

For small velocities one can get a linearized expression of this force : $f(v) = -\gamma v$ yielding a friction coefficient γ. Because of the important friction this type of cooling has been called 'optical molasses'. If one would change the frequency of the laser to positive ("blue") detuning δ, one would get a heating process for the atoms (increasing their velocities). The limit of such a cooling process is given by the quantum fluctuations of the forces and depends on the recoil velocity $\frac{\hbar k}{m}$. A diffusion coefficient D for the velocities can be used to describe these fluctuations and its balance with the friction sets the equilibrium temperature of the atoms.

The spatial dependence required to confine atoms in a MOT is obtained using appropriate polarizations of the counterpropagating lasers and applying a magnetic field gradient. This results in a force similar to eq. (1), replacing kv by $kv + \mu z$, where μ describes the Zeeman shift of the transitions. A linear expansion around $v = 0$ and $z = 0$ yields a force corresponding to a damped harmonic oscillator :

$$f(v) = -\gamma v - \kappa z \qquad (2)$$

The size of the atom cloud is determined by the balance between kinetic and potential energy

$$\frac{1}{2}\kappa < z >^2 = \frac{1}{2} m < v >^2 = \frac{1}{2} k_B T \ . \qquad (3)$$

In this regime, the size of the MOT depends on the trap paramters such as detuning, intensities and magnetic field gradient. Adding more atoms to the trap will not increase the spatial extend of the trap, but the spatial density n will be proportional to the atom number N. If the size and the temperature of the MOT would be independent of the number of atoms, one would expect a transition to a degenerate gas for $N \approx 10^{11}$ (at a MOT size of $200\mu m$ and a temperature of $10\mu K$). As we will see below, the ultimate goal of Bose-Einstein condensation could not be reached via this route because of limitations due to multiple light scattering.

LONG RANGE INTERACTION BY MULTIPLE SCATTERING : MOT SIZE

The first manifestation of collective effects in a MOT is the increase of its size with the number of trapped atoms N. If N is sufficiently small such that collective effects can be neglected, the size of the trap is typically given by the balance between the kinetic energy and the potential energy of a linear harmonic oscillator (temperature limited regime). When the number of trapped atoms is increased the light scattering by the atoms starts to modify the light induced forces. Two competing effects have to be considered: a shadow effect, which tends to yield an additional compression of the trap and a repulsive radiation pressure effect, which tends to increase the size of the cloud [5]. The shadow effect is due to attenuation of the laser light inside the cloud which yields an effective attraction between two particles. Similar effective attractive interactions occur in a variety of situations (e.g. Van der Waals attraction) and are reminiscent of the Lesage model of gravitation based on screening. In the framework of cold atom traps, this shadow effect has been introduced in [6] and can be interpreted as a spring constant which is modified by absorption. Note that one intriguing and important feature of this screening is that is yields a long range interaction as for gravity.

On the other hand, the compression of the cloud in counteracted by collective effects due to multiple scattering of the irradiated laser light. Rescattering of light between two atoms produces a repulsive interaction as a photon scattered from one atom tends to push away the second atom. The stability of the atomic cloud is determined by the relative importance of the two effects. In the typical MOT situation, the repulsion unfortunately dominates, such that the cloud size increases with increasing atom number.

Let us now study the light induced interaction between two atoms at r_1 and r_2. Neglecting again the dipole forces, the force on the induced dipole at r_2 is given by: $\overrightarrow{F}_{rad} = \alpha'' k \overrightarrow{e}_r \frac{I_{scat}}{c}$ where I_{scat} is the scattered intensity incident on atom 2. A spherically symmetric total power P emitted by atom 1 ($P = \sigma_{at} I_{inc}$, with $\sigma_{at} = \frac{3\lambda^2}{2\pi} \frac{\frac{\Gamma}{2}}{\delta^2 + \frac{\Gamma^2}{4}}$) implies $4\pi r^2 I_{scat} = P = \sigma_{at} I_{inc}$, resulting in a Coulomb-like atomic interaction

$$\overrightarrow{F}_{rad} = \overrightarrow{e}_r \frac{(\sigma_{at})^2}{4\pi r^2} \frac{I_{inc}}{c} \ . \tag{4}$$

RELATION TO NON-NEUTRAL PLASMAS

For direct comparison with Coulomb-interacting charges we define an effective charge q_{eff} such that $\overrightarrow{F}_{Cb} = q_{eff}^2 \overrightarrow{e}_r / (4\pi\varepsilon_0 r^2)$ leading to

$$q_{eff}^2 = \varepsilon_0 \, (\sigma_{at}^{res})^2 \, \frac{I_{inc}}{c} \tag{5}$$

For typical values $I_{inc} = 1 mW/cm^2$ and $\sigma_{at}^{res} = 0.3 10^{-8} cm^2$ we obtain

$$q_{eff}^{res} = 10^{-3} e \tag{6}$$

where e denotes the electron charge. Note that this is the maximum value for resonant scattering. A finite laser detuning reduces the effective charge according to $q_{eff} = q_{eff}^{res}/(1+4(\delta/\Gamma)^2)$, which yields $q_{eff} \simeq 10^{-5}e$ for $\delta = -3\Gamma$.

In order to work to relation to plasma physics systems we may evaluate various effective plasma parameters. A total interaction energy $\frac{Nq_{eff}^2}{4\pi\varepsilon_0 R}$ larger than the kinetic energy $k_B T$ of the particles leads to an increased size $L = 2R$ of the magneto-optical trap when the number N of atoms exceeds 10^5. At this point collective effects start to become important, just like in a trapped plasma. The standard definition of a plasma requires the Debye length to be less then the size of the system. From the effective charge and for typical atom temperatures we find a Debye length on the order of a few $100\mu m$, well below the typical size of a large MOT. Thus, our laser-cooled atom cloud may indeed be considered as a trapped non-neutral plasma. Moreover, in our experiments the corresponding plasma frequency is $\omega_p = \sqrt{nq_{eff}^2/\varepsilon_0 m} \approx 200Hz$, of the same order as typical relaxation rate of the atomic positions in our MOT($\approx 50Hz$). The interplay of these frequencies might be worthwhile to study.

An important advantage of the light induced collective interactions is that the effective charge q_{eff} depends on experimental control parameters, allowing for a engineering of the effective charge which can be varied by orders of magnitude. An important quantity is the ratio between the Coulomb interaction and the kinetic energy, i.e. the Coulomb coupling parameter. Let us evaluate the possibility of creating a strongly coupled state or even Coulomb crystal with neutral atoms. The phase of one-component plasmas is uniquely determined by the Coulomb coupling parameter, Γ. When Γ exceeds ≈ 174, molecular dynamics simulations predict a transition from a liquid-like to a solid state – a Coulomb crystals. For ions in traps where the typical densities are 10^8-10^9 cm^{-3}, laser cooling to temperatures of a few milli-Kelvin is sufficient for obtaining such crystals. We estimate the Coulomb coupling parameter to be smaller than unity in our system, excluding any crystallization effect. For neutral atoms in a MOT, Coulomb crystallization seems extremely difficult to reach, but the regime $\Gamma > 2$, were liquid-like behavior sets in, should, however, be realizable in experiment. However, it might be possible to use the high phase space densities of a BEC and thus study strongly coupled plasma in the degenerate regime as expected in neutron stars and white dwarfs.

MOT SIZE AND DENSITY LIMITS

As stated above the size of the MOT is determined by the balance between the shadow effect and multiple scattering. One can show that for small optical thickness[5], the repulsive interaction due *elastic* multiple scattering exactly compensates in 3D the shadow effect. The reason for the repulsion to dominate over the compression arise inelastic scattering. For typical MOT parameters the atomic saturation is not small, and the scattered light has a different frequency, following the so-called so-called Mollow triplet spectrum. One thus has to compare the efficiency of absorption of light from the incident lasers σ_L to that of reabsorption of inelastically scattered light $< \sigma_R >$, averaged over the spectrum of the scattered light. The complete calculation of $\langle\sigma_R\rangle - \sigma_L$

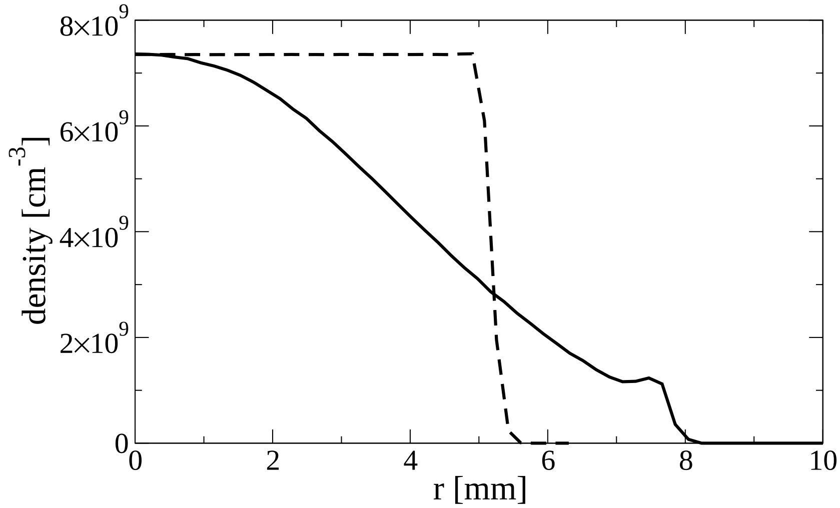

FIGURE 1. Calculated density profile of the trapped atoms, including (solid line) and excluding (dashed line) the effect of position dependent effective charges.

is somewhat tedious, but can be done following [7]. Here we only discuss the limit of $\delta \gg \Omega \gg \Gamma$ which yields $\langle\sigma_R\rangle - \sigma_L = \frac{3\lambda^2}{2\pi}\frac{\Omega^2}{8\delta^2}$ and hence

$$\frac{\langle\sigma_R\rangle - \sigma_L}{\sigma_L} = \frac{\Omega^2}{8\delta^2(1+4\delta^2)} \ll 1\ . \tag{7}$$

As a result, the model proposed in [5] predicts a constant, step-like density profile $n(r)$ of the atoms in the MOT resulting in a trap size L scaling as $L \sim N^{1/3}$.

This scaling law has been observed in several experiments for atom numbers ranging from 10^6 to 10^8. In our group, we have realized a very large MOT, with the primary goal to study multiple scattering of light in atomic vapors and its possible road towards Anderson localization [2]. We are able to trap as many as 10^{10} atoms, making this trap entering a relatively little explored regime. The observed density profile in our large MOT strongly deviates from a step-like profile, being closer to a Gaussian shape even in the multiple scattering limited regime.

To investigate this issue, we have a developed a kinetic model based on a particle-in-cell treatment of the atom-atom interaction. In addition to the simple discussion provided above, this more sophisticated approach also accounts for the influence of the laser attenuation and the Zeeman shift on the atomic interaction. This results in an effective charge which depends locally on the atom position (due to the Zeeman shift) but also non-locally on the entire density distribution (due to the laser attenuation). As shown in fig.1, our model indeed reproduces the observed, non-constant density profile, and demonstrates the importance of the position dependence of the effective charge. A decreasing atom number leads to a transition to a step-like profile, which is found in simpler descriptions. Note that these additional effects cause deviations from a pure Coulomb-like interaction and can also violate Newton's third law of motion (actio=reactio). The consequences of such a non-local space dependence of the charges for this plasma-like systems are currently investigated.

For completness, let us mention that interference between the scattered and the incident light can lead to long range dipole induced forces, reminiscent of forces at the origin of optical binding. We have neglected these effects, as well as higher order terms in screening, which tend to reduce the shadow effect and screen the binary repulsion as well.

SELF-SUSTAINED OSCILLATIONS IN A LARGE CLOUD OF COLD ATOMS

In this section we describe a new regime of large MOTs, which appears for very large sizes, when the non linearities of the trapping and cooling forces can no longer be neglected. Several groups seem to have observed similar features, but without detailed investigations and understanding there is, to our knowledge, no published report of the instabilities which we will describe below. Details of this work has been published in [9, 10].

Our MOT collects Rb85 atoms from a dilute vapor using six large independent laser beams (beam waist $4cm$, power per beam $P = 30mW$) thus avoiding intensity imbalance due to the opacity of the cloud. A magnetic field gradient ($\nabla B = 10G/cm$) is applied to generate the magneto-optical restoring force of the trap. Under standard operating conditions, the trapping lasers are detuned from the $F = 3 \rightarrow F' = 4$ transition of the $D2$ line by $\delta = -3\Gamma(\Gamma/2\pi = 6MHz)$. A repumping laser on the $F = 2 \rightarrow F' = 3$ of the $D2$ line is used to control the total number of atoms. We thus obtain a MOT with up to $N = 10^{10}$ atoms (diameter $L = 5mm$, $T = 100\mu K$). The size and shape of the cloud is monitored by imaging the MOT fluorescence on a cooled CCD. The optical thickness b of the cloud at the trapping laser frequency is deduced from the width of the transmission spectrum, taking into account the linewidth (2MHz) of the laser. To obtain a time-resolved information on the size or local density of the MOT, we also image a portion of the cloud on another photodiode. The most striking observation is the onset of spontaneous self-sustained oscillation for a sufficiently large number of atoms.

This collective behaviour can be observed during loading of the MOT, when we switch on the MOT at $t = 0$ and monitor the time evolution of the fluorescence from a portion of the MOT. This partial fluorescence signal roughly corresponds to the number of atoms in the observed region. Starting from $N = 0$ at $t = 0$, the trap is filled on a time scale $\tau = 1.45s$, determined by the ambient Rb pressure. Below a critical number of atoms N_{th}, the size of the atom cloud increases but no specific dynamical behavior is observed. Above the threshold N_{th} the cloud appears to switch to a breathing mode characterized by periodic oscillations in the partial fluorescence signal. We have investigated this breathing mode in detail and we have developed a qualitative model for this bifurcation. Some information can be gained by having a closer look at the shape of the oscillations above the threshold. Fig.2 shows an example where we detect the fluorescence from the center of the MOT. As the MOT is expanding, the density at the center of the cloud is decreasing whereas the compression phase of the cloud is associated to the increasing part of the fluorescence. A high contrast modulation of the center fluorescence is observed in this experiment. The oscillations are quite periodic, but with an asymmetric shape: a fast expansion of the cloud followed by a slower

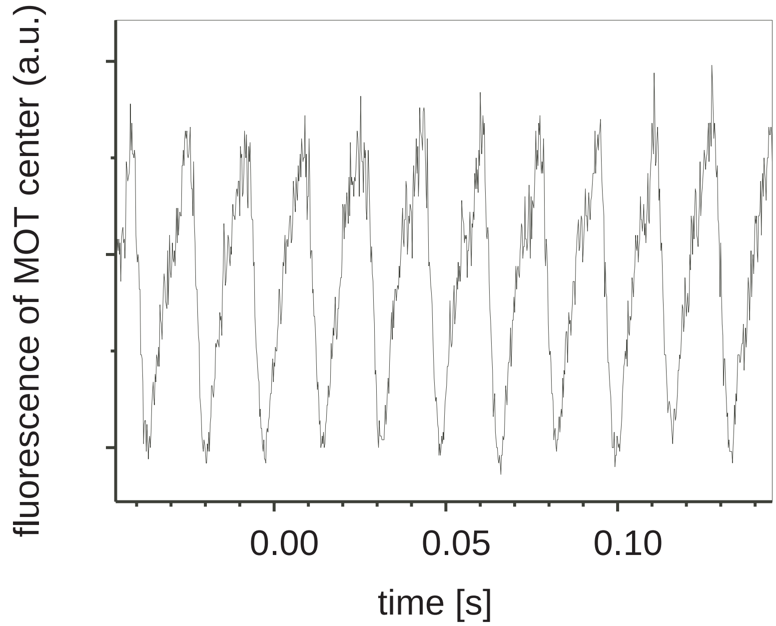

FIGURE 2. Fluorescence of the MOT center. A fast expansion of the cloud (reduction of the fluorescence signal) is followed by a slower compression phase, yielding a 'saw-tooth' shape.

compression phase. A fast Fourier transform of such data is used to characterize the frequency and the amplitude of the oscillation. We have also checked that the number of atoms is constant during the oscillation (at a few percent level, i.e. at the same level as in the stable MOT regime). This contrasts to instabilities observed with a MOT using 3 retro-reflected mirrors, where the long range repulsion can be neglected and the imbalance effect coupled to the total number of atoms dominates the dynamics of the center of mass of the cloud [11].

Investigating the MOT at the instability threshold by varying 2 of the various control parameters of the experiment, we can map a phase diagram such as shown in Fig. 3 (full squares). As can be seen, the overall behavior is unstable when the trapping laser frequency is brought within roughly one natural width from resonance. The solid line corresponds to the theoretical prediction presented below [9]. We also measured, along the threshold boundary, the size of the cloud, the number of atoms, and the peak attenuation of the trapping laser. Although both the number of atoms and the size do vary during these measurements, we systematically found an optical thickness $b \approx 1$ at the instability threshold. However, this is clearly not a sufficient condition for the onset of instabilities, since $b = 1$ is also observed in the stable region. As a further characterization of the threshold, we have checked that the amplitude of the oscillation continuously grows from zero as the control parameter (detuning and number of atoms in the experiments performed) crosses the threshold value. Also a Fourier analysis of the signal across the threshold showed that the instability start at a non zero frequency, which is closely related to the trap oscillation. Furthermore, no hysteresis has been observed despite explicit investigation (see Fig.4). All these findings are consistent with

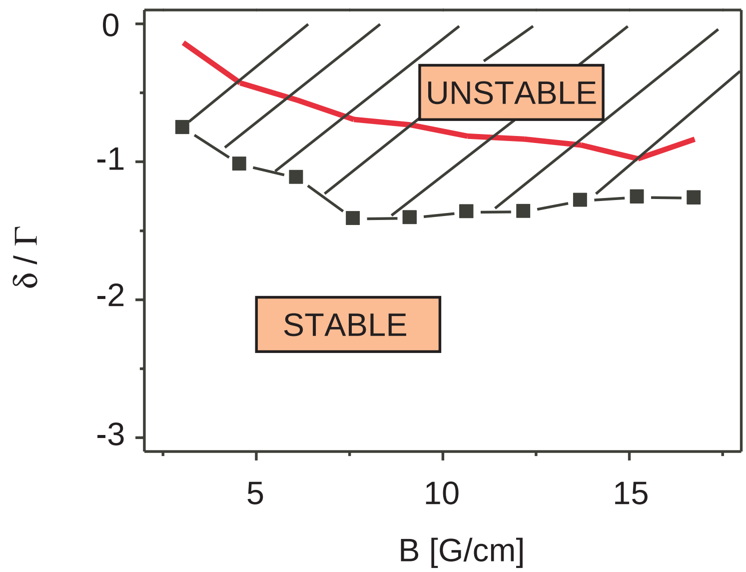

FIGURE 3. Phase diagram of self-sustained oscillation : a $(\delta, \nabla B)$ cut in parameter space shows the separation between the stable (for larger detuning δ) and the unstable regime: experimental threshold values (squares) and 1-zone model prediction (solid line).

a supercritical Hopf bifurcation.

The most simple model which exhibit an instability threshold is a 1-zone model [12]. In this model we assume a homogeneous density, where the size of the cloud L is related to the density n via the total number of atoms N as $n = N/L^3$. The dynamics along one symmetry axis (Ox) of a probe particle located outside of the cloud (at position $x \sup L/2$ from the trap center, with a velocity v) is then governed by the force :

$$f(x,v) = \hbar k \frac{\Gamma}{2} \frac{I_{inc}}{I_{sat}} \frac{e^{-b}}{1+\frac{4(\delta - kv - \mu x)^2}{\Gamma^2}} - \hbar k \frac{\Gamma}{2} \frac{I_{inc}}{I_{sat}} \frac{1}{1+\frac{4(\delta + kv + \mu x)^2}{\Gamma^2}} + \eta \hbar k \frac{\Gamma}{2} \frac{I_{inc}}{I_{sat}} \qquad (8)$$

This expression relies on the low intensity Doppler model for the magneto-optical force (incident on-resonance saturation parameter : s_{inc}). The first term in this expression is the attenuated force of the laser passed through the cloud (with the corresponding Zeeman shift μx and Doppler shift kv), the second term corresponds to the non attenuated force of the laser propagating in the opposite direction (with opposite Zeeman and Doppler shift). The last terms is the sum of all repulsive interactions which, using Gauss theorem, yields an $1/r^2$ repulsion for the probe particle. Here η corresponds to the ratio between the absorption cross section of the incident laser frequency and the inelastically rescattered photons [5]. We now apply this model at the edge of the cloud ($x_0 = L/2$). A linear stability analysis, using $x = x_0 + \delta x e^{i\omega t}$ around the fixed point $F(x_0, v=0) = 0$, yields the threshold condition for an instability :$\Im(\omega) = 0$. We find that for our experimental parameters the threshold is given with a good approximation by :

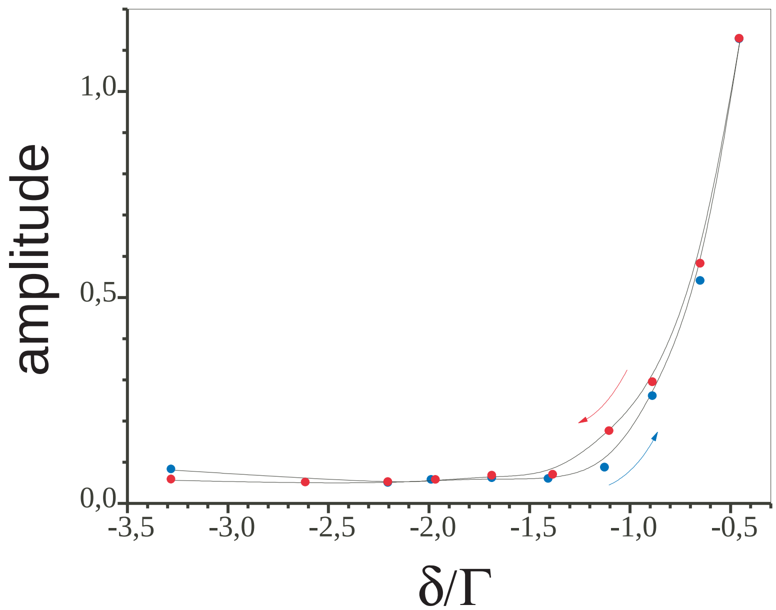

FIGURE 4. Signature of a super critical Hopf bifurcation : amplitude of the oscillation from a fast Fourier analysis : a slow scan across the transition shows no hysterisis

$$\delta - \mu x = 0 \tag{9}$$

In contrast to standard MOT operation (where $\mu x \ll \|\delta\|$) the edge of the cloud is now exploring the non linear part of the magneto-optical forces. A spatial increase in the MOT size is there less counteracted than at the center of the MOT. The threshold condition eq. 9 is associated to a negative local friction : the MOT operation for increasing number of atoms is thus evolving from an overdamped to an underdamped oscillator before switching to an unstable oscillator. The oscillation frequency $\omega_{osc} = \Re(\omega)$ at threshold is given by the natural oscillation frequency of the oscillator and does not undergo any spectacular change across the threshold. In order to confront this simple model to the experiment, we compute the threshold value for the detuning δ using the control parameter ∇B and the measured value for the size of the cloud $x_0 = L/2$ and its optical thickness b at the threshold. The result gives the correct order of magnitude for the instability threshold. This model can also be used to estimate the number of atoms needed to reach the instability threshold and explain why usual MOTs are stable. Only MOTs with large number of atoms $N > 10^9$ and operated at small detuning can present the instabilities discussed in this paper.

Our more sophisticated kinetic model [10] revealed that the instability is driven by the radiation pressure which changes its sign at $r = \delta/\mu$, consistent with the simple discussion given above. Moreover, we find that stable oscillations can only exists if the repulsive interaction pressure decreases faster than the confining force at large distances. This requirement is provided by the space dependent effective charges, discussed above, underlining the importance of new effects to understand the regime of large laser-cooled atom clouds discussed in this work.

CONCLUSION

We have presented a mecanism of Light Mediated Long Range Interaction between neutral atoms which lead to a well known increase of the MOT size and give an anaolgy to non neutral plasma phsyics. In addition we observe an instability based on a competition between the screened confining forces and size dependent repulsion which cannot be obtained without including the long range repulsion. Recently we have heard about similar collective instabilities in ion traps [13] and it would be interesting to analyze in more detail possible analogies based on non linear resonances. Future possibilities for this experiments include the forced oscillation regime, the spectroscopy of excitation modes in this system, wave propagation based on collective effects and measurements of the spatial pair correlation function. Feedback mechanisms might allow to study the possibility of stabilization of a large cloud of interacting particles and precursors to the instability could be looked at in the noise spectrum, velocity or space correlation functions. If the degenerate regime could be reached (using a Bose Einstein Condensate) a mean field theory based on binary collisions as in usual Gross Pitaevskii equation will not be valid due to the long range interaction, connecting this system to strongly correlated quantum systems.

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