Nondestructive detection of atoms bouncing on an evanescent wave

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A cloud of atoms reflecting from an evanescent-wave mirror produces a phase shift on the beam producing the evanescent wave. This shift can be used to detect the passage of the atoms. We give a physical discussion of this process and estimate the phase shift. The shift can be greatly increased by using a resonant planar waveguide. We also estimate the signal-to-noise ratio and show how to optimize the experimental parameters of such a measurement. Using the waveguide we find a signal-to-noise ratio of order of 300 in a realistic experimental situation.

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I. INTRODUCTION

In the rapidly developing field of atom optics [1,2], high quality atomic mirrors play a key role in many experiments. For example, a recent experiment observed more than ten successive bounces on a concave evanescent-wave mirror [3]. In order for such an experiment to exhibit the wave nature of the atoms, it is important to avoid processes such as spontaneous emission, which result either in atoms escaping from the cavity or the loss of the de Broglie wave coherence.

To observe the motion of the atoms, previous experiments have used resonant probe laser beams and monitored the atomic fluorescence or absorption [3]. If the probe laser is strong this probing significantly heats the atoms, causing them to be lost. By using a very weak probe this heating can be reduced, but spontaneous emission is still present. To eliminate this effect, one could use an off-resonant probe laser. This laser would experience a phase shift due to the atomic dispersion, which could be detected through interference with a reference beam [4,5]. Although the dipole force can still modify the atomic motion, spontaneous emission can be rendered negligible by going far away from the atomic resonance. Taking this idea even further, one can use a standing wave in a Fabry-Pérot cavity as the probe. Several authors have shown [6] that a measurement of the phase shift induced by the atoms can be used to resolve the atoms' position along the standing wave to much less than the optical wavelength.

In this paper, we propose a similar measurement in which one measures the phase shift of the laser producing the evanescent-wave mirror. This method has the virtue of being nondestructive in the sense that no additional light is introduced into the experiment. We simply propose to gather information that is already present in an evanescent-wave mirror. Like the proposals of Refs. [6], our phase measurement can in principle resolve the atoms' position to less than the optical wavelength, or more precisely in our case, to less than the decay length of the evanescent wave.

The paper is devoted to a physical discussion of the interaction of a cloud of atoms, that bounce on an evanescent wave, and the laser producing that wave, which is phase shifted by the atoms' presence. The experimental variables affect the phase shift in a complicated way, and we try to give a clear physical description that will guide us in choosing the experimental parameters. We limit ourselves to treating the atoms as two-level systems with classical trajectories, and estimate the signal-to-noise ratio. We will show that a resonant waveguide structure that we have developed to enhance the evanescent wave results in a large increase in the sensitivity of this method, rendering it feasible under realistic experimental conditions. The waveguide is somewhat analogous to the high finesse cavities used in Refs. [6].

The possibility of detecting the atom position without apparent perturbation raises interesting questions about the possible back action of the measurement on the atomic de Broglie wave, and more generally on the "quantum nondemolition" [7] aspects of such a measurement. We will touch qualitatively on this point in the conclusion, and refer to the fully quantum-mechanical treatment of these issues presented in Ref. [8].

II. PRINCIPLE OF THE METHOD

The principle of the evanescent-wave mirror has been described elsewhere [9,10], but we remind the reader of some of its essential features. We consider a laser beam of frequency ω_L totally internally reflected at a glass-vacuum interface (Fig. 1). The angle of incidence is θ_1, and the index of refraction is n_1. We restrict ourselves to the limit of large detuning and low saturation, in which spontaneous emission during the bounce is negligible, and we neglect the van der Waals interaction between the atoms and the surface. We also assume the atoms are two-level systems characterized by a resonance frequency ω_a, a linewidth Γ, and a saturation intensity I_{sat}. In this case the trajectory of the atoms along the z axis normal to the plane of the reflection surface is governed by a potential,

$$V(z)=\frac{\hbar\Gamma}{8}\frac{\Gamma}{\Delta}\frac{I_{ev}}{I_{sat}}\exp(-2\kappa z). \tag{1}$$

Here, $\Delta=\omega_L-\omega_a$ is the detuning from resonance, and $\kappa=(\omega_L/c)\sqrt{n_1^2\sin^2\theta_1-1}$ is the inverse decay length of the evanescent wave. The effective intensity of the evanescent wave I_{ev} is related to the evanescent-wave electric field E_{ev} by $I_{ev}=\frac{1}{2}\epsilon_0 c|E_{ev}|^2$. The quantity I_{ev} is also related to the incident laser intensity I_1, in the case of TE (or s) polarization, by

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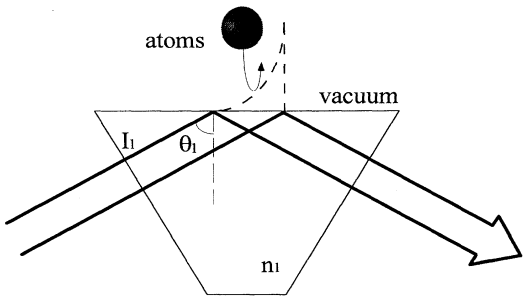

FIG. 1. Principle of the detection scheme. A cloud of atoms is reflected by the evanescent wave that is created by the totally reflected laser beam (denoted by the arrow) of intensity I_1. The dotted curve denotes the effective intensity profile of the evanescent wave. The change in the index of refraction above the interface due to the atoms causes a slight change of the phase of the reflected laser wave.

$$I_{ev}=T_1 I_1=\frac{1}{n_1}\frac{4n_1^2\cos^2\theta_1}{n_1{}^2-1}I_1. \tag{2}$$

Specifically, the quantity I_1 is the intensity incident on the total internal reflection interface. If the input face of the prism is antireflection coated and normal to the input beam, I_1 is also the intensity of the incident laser outside the prism. Our definition of the coefficient T_1 is the product of the usual Fresnel coefficient for total internal reflection and a factor $1/n_1$, which appears in the expression relating I_1 to E_1, the electric field inside the prism of index n_1.

The atoms change the index of refraction of the region above the interface when they bounce on the evanescent wave. For an atomic density distribution $\rho_{at}(z)$ and in the limit of large detuning and low saturation, the index of refraction is

$$n_{at}(z)=1-\frac{3}{16\pi^2}\frac{\Gamma}{\Delta}\rho_{at}(z)\lambda^3. \tag{3}$$

Because of this index of refraction, the presence of the atoms changes the phase of the reflected laser beam. For a TE polarized wave, the corresponding phase shift is [8,11]

$$\varphi=-\frac{3}{2\pi}\frac{n_1\cos\theta_1}{n_1{}^2-1}\frac{\Gamma}{\Delta}\lambda^2\int_0^\infty \rho_{at}(z)\exp(-2\kappa z)dz. \tag{4}$$

In the case when $n_{at}-1$ is small, this formula can be derived by using the known value of the phase shift upon total internal reflection at an interface [12]. The nonuniform atomic distribution along z can be modeled by successive slabs that are not in contact with the glass, resulting in the exponential factor in the integral.

Unlike for a thermal atomic vapor, the atomic density of atoms bouncing above the surface is not uniform. Atoms with a well defined velocity v_{in} are reflected at the turning point z_b defined by

$$\frac{1}{2}Mv_{in}{}^2=\frac{\hbar\Gamma}{8}\frac{\Gamma}{\Delta}\frac{I_{ev}}{I_{sat}}\exp(-2\kappa z_b). \tag{5}$$

The atomic density is therefore zero between the interface and the turning point. Above the turning point, the density is found using the velocity as a function of z and the conservation of the atomic flux: $\rho(z)v(z)=\rho_{in}v_{in}$, where ρ_{in} is the density far from the mirror. This yields

$$\rho_{at}(z)=\rho_{in}\frac{2}{\sqrt{1-\exp\{-2\kappa(z-z_b)\}}},\quad \text{for } z\geq z_b. \tag{6}$$

Note the kinematic compression due to the decrease of the velocity, and the factor of 2 due to the fact that the atom flux is folded over on itself on reflection.

Substituting this expression into Eq. (4), we find for the laser phase shift due to the bouncing atoms

$$\varphi=-\frac{3}{\pi}\frac{n_1\cos\theta_1}{n_1{}^2-1}\frac{\Gamma}{\Delta}\frac{\lambda^2}{\kappa}\rho_{in}\exp(-2\kappa z_b). \tag{7}$$

As mentioned above, the exponential reduction factor is due to the absence of atoms between the turning point and the surface.

III. DISCUSSION OF THE SENSITIVITY

In order to evaluate the sensitivity of the method, we will estimate the signal-to-noise ratio for the laser phase shift measurement at a given incident atomic density ρ_{in} and velocity v_{in}. We consider the index of refraction of the glass n_1 and the incidence angle θ_1 as fixed parameters, so that the laser phase shift φ is only a function of the laser detuning Δ and of the evanescent-wave intensity at the interface I_{ev} [remember that the position of the turning point z_b depends on both these quantities; see Eq. (5)]. Substituting Eq. (5) into the expression [Eq. (7)] of the phase shift, we obtain

$$\varphi=-\frac{12}{\pi}\frac{n_1\cos\theta_1}{n_1{}^2-1}\frac{Mv_{in}{}^2}{\hbar\Gamma}\frac{I_{sat}}{I_{ev}}\frac{\lambda^2}{\kappa}\rho_{in}. \tag{8}$$

This equation demonstrates the curious result that for v_{in} and ρ_{in} fixed, the phase shift is inversely proportional to the evanescent-wave intensity, and is independent of the detuning used to bounce the atoms. This can be understood as follows: if the intensity is increased, the atoms' turning point is further away from the interface so they are less strongly coupled to the laser field and produce a smaller phase shift. Decreasing the detuning has the same effect, but in this case the decrease in the coupling strength is compensated by an increase in the susceptibility of the atoms. Note, however, that the intensity cannot be chosen arbitrarily small, because it must be large enough that the turning point z_b be positive. This condition imposes a minimum value for I_{ev}, which depends on the detuning and the velocity v_{in} [Eq. (5) with $z_b=0$].

To detect the phase shift [Eq. (8)], we envision a measurement in which the reflected laser beam and a reference interfere (see Sec. V). We consider an incident cloud of atoms of extent l along the z axis. This cloud interacts with the evanescent wave for a time $\tau_{meas}=l/v_{in}$ (l is large compared to the extent κ^{-1} of the evanescent wave), and we take this time as the duration of the measurement. If we assume that

the measurement is limited only by shot noise, we may write the standard deviation of a phase measurement as

$$\delta\varphi_N=\frac{1}{\sqrt{N}}=\sqrt{\frac{\hbar\omega}{I_1 S\tau_{meas}}}=\sqrt{\frac{T_1\hbar\omega}{I_{ev}S\tau_{meas}}}. \tag{9}$$

Here N denotes the number of detected photons, which is related to the intensity I_1 and to the cross section S of the laser beam at the interface. We assume that the area $S/\cos\theta_1$ matches that of the atomic cloud that falls on the mirror, and that the detection efficiency is unity.

The signal-to-noise ratio of a single phase measurement is given by $|\varphi|/\delta\varphi_N$. From Eqs. (8) and (9) we see that the intensity must be chosen as small as possible to maximize this ratio. The corresponding value of I_{ev} is obtained by taking $z_b=0$ in Eq. (5). Since this value depends on Δ, we can then express the signal-to-noise ratio as a decreasing function of the detuning.

To fix the detuning, we impose the value of the probability of spontaneous emission per bounce. In the low saturation, off-resonant regime, this probability is related to the detuning by [3,10]

$$P_{sp}=\frac{\Gamma}{\Delta}\frac{Mv_{in}}{\hbar\kappa}. \tag{10}$$

Finally, using this expression for the detuning, we obtain the signal-to-noise ratio for a measurement of the phase shift due to the atoms bouncing,

$$\frac{|\varphi|}{\delta\varphi_N}=\sqrt{\frac{3}{\pi}\frac{n_1}{n_1^2-1}}\sqrt{P_{sp}\frac{\lambda^2}{\kappa}Sv_{in}\tau_{meas}\rho_{in}^2}, \tag{11}$$

where we have used $I_{sat}=(\pi/3)(\Gamma/\lambda^2)\hbar\omega$ for the saturation intensity.

To give a numerical example, let us assume a cloud of Rb atoms ($\lambda=780$ nm , $\Gamma/2\pi=6$ MHz), 1 mm on a side with a density of 10^{10} cm^{-3}, falling at a velocity of 50 cm/s on an evanescent-wave mirror. This is easily possible with current atom cooling techniques. Under these conditions, the duration of the measurement τ_{meas} is 2 ms . We take $\theta_1=60°$ and $n_1=1.9$ (this is the index used in Ref. [13]), and we choose the spontaneous emission probability to be $P_{sp}=10^{-2}$. This requires a detuning of 40 GHz (6500 Γ), and the minimum laser intensity to reflect the atoms is about 600 W/cm^2 ($3\times10^5 I_{sat}$). Finally, Eq. (8) gives a phase shift of 3×10^{-8} rad and Eq. (11) gives a signal-to-noise ratio of about 4.

Although our example shows that such a detection scheme is possible in principle, our estimate is very optimistic, because with a laser power of approximately 3 W, a shot-noise limited measurement with an integration time of 2 ms is likely to be very difficult. In addition, our assumption of $z_b=0$ is not realistic, since the presence of van der Waals interaction prevents us from adjusting the intensity such that the atoms bounce arbitrarily close to the prism. In the example given above the van der Waals interaction remains negligible if $z_b\cong\kappa^{-1}$ (about 100 nm). This means that the above estimate of the phase shift is reduced by a factor of e^2 and the signal-to-noise ratio by a factor of e. In the next section we will show how a resonant structure can be used to enhance the signal-to-noise ratio of the measurement by a large factor.

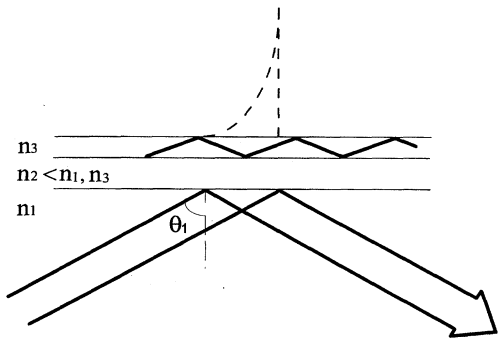

FIG. 2. Schematic of the waveguide structure used to enhance the intensity of the evanescent wave. The incident laser excites a mode in the waveguide (index n_3) by evanescent-wave coupling. The thickness of the waveguide is of the order of 100 nm, and that of the gap layer (index n_2) several hundred nm. This waveguide acts analogously to a Fabry-Pérot build-up cavity, enhancing the evanescent-wave intensity as well as the laser phase shift due to a small change in the index above the waveguide.

IV. ENHANCING THE SENSITIVITY WITH A WAVEGUIDE LAYER

Reference [13] demonstrates that the intensity of the evanescent wave can be enhanced by several orders of magnitude by a dielectric waveguide. The waveguide layer is separated from the prism base by a low index gap layer (Fig. 2). The incident laser couples by optical tunneling through the gap layer into the waveguide. If the component of the laser wave vector along the interface is matched to that of a mode of the waveguide, the electric field in the waveguide, and also that of the evanescent wave, is resonantly enhanced. The waveguide layer acts as a build-up cavity analogous to a Fabry-Pérot cavity, with a 100% reflection output mirror, and a low transmission input mirror corresponding to the evanescent-wave coupling across the gap. In the regime of weak coupling and negligible losses in the waveguide, one can show [13,14] that, as in a Fabry-Pérot cavity, the waveguide intensity enhancement (build-up factor) increases with decreasing coupling strength. Close to the waveguide resonance, the evanescent-wave intensity, defined in analogy with Eq. (2), has a Lorentzian dependence on angle

$$I_{ev}=T_{14}I_1=T_{res}\frac{\Delta\theta_1^2}{(\theta_1-\theta_{res})^2+\Delta\theta_1^2}I_1, \tag{12}$$

with a resonance width $\Delta\theta_1$ inversely proportional to the resonant enhancement factor T_{res}.

As in the case of a Fabry-Pérot cavity, the sensitivity to small phase shifts increases by the same factor as the intensity enhancement. To see the effect on the phase Φ of the reflected laser wave (note that Φ refers to the total phase on reflection, while φ refers to the shift due to the atoms), we write the amplitude reflection coefficient

$$r_{14}=r_{12}\exp(-i\Phi), \tag{13}$$

where r_{12} is the amplitude reflection coefficient at the prism-gap interface in the absence of the waveguide. The modulus of r_{12} is unity, and its phase varies slowly as a function of θ_1. The phase Φ undergoes a rapid variation across resonance

$$\tan\frac{\Phi}{2}=\frac{\Delta\theta_1}{\theta_1-\theta_{res}}, \tag{14}$$

with a slope that is inversely proportional to $\Delta\theta_1$ and thus proportional to T_{res}.

When the index of refraction above the waveguide is modified by the presence of the atoms the resonance angle θ_{res} is changed, and one can show that the phase shift φ_{res} of the laser beam due to the atoms is enhanced in comparison to that of a bare prism φ [Eq. (4)] by

$$\varphi_{res}=\frac{T_{res}}{T_1}\varphi. \tag{15}$$

In the comparison displayed in Eq. (15), note that we have assumed that the atomic trajectories are held fixed, that is, we choose I_{ev} to be small enough that the atoms bounce close to the interface. Therefore our enhancement technique not only permits, but requires us to use an intensity smaller by a factor of T_{res}/T_1 than in the case of a bare prism. Then, assuming the only limitation to be laser shot noise, the standard deviation of a phase measurement $\delta\varphi_N$ increases by a factor of $\sqrt{T_{res}/T_1}$ compared to Eq. (9), and thus the signal-to-noise ratio increases by a factor of $\sqrt{T_{res}/T_1}$. We have succeeded in producing prisms with an enhancement T_{res}/T_1 of about 3000 [14]. Therefore using the numerical example cited above, we expect a phase shift $\varphi_{res}=10^{-4}$ rad and a signal-to-noise ratio of about 300.

V. SOME EXPERIMENTAL CONSIDERATIONS

We have shown that if we can achieve or at least approach the shot-noise limit, we will be able to detect the laser phase shift caused by the bouncing atoms. Of course various sources of technical noise present serious obstacles to achieving the ultimate sensitivity of the type of experiment we have described. Here we will discuss some strategies for reducing the technical noise. A possible detection scheme is shown in Fig. 3. We consider a polarization interferometer in which a beam containing both TE and TM field components is incident on the resonant structure of the prism. We suppose that only the TE polarization is resonant with the waveguide. Because of the waveguide resonance, the passage of the atoms causes a phase shift in the TE component, which is much larger than that of the TM component. (Note that because of the weak tunneling across the gap, the strength of the TM component is not only nonresonant but suppressed.) Therefore we introduce an analyzer after the prism, consisting of a retarder and a polarizing beam splitter, which divides the TM and TE amplitudes each into equal portions in each port of the analyzer. This causes the TM and TE waves to interfere, and the difference in the detected intensities at each output port is related to the phase shift. Subtracting the two intensities helps to reduce the effect of the intensity noise of the laser.

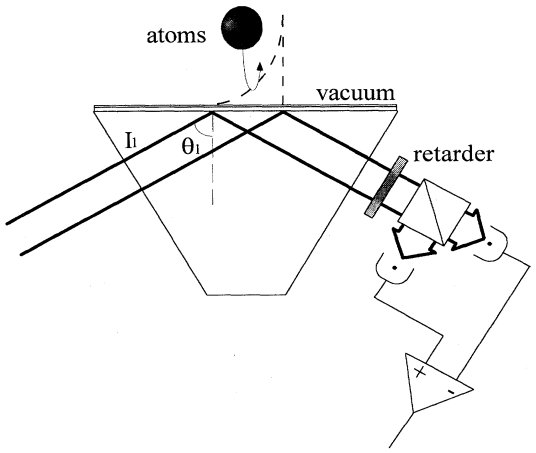

FIG. 3. Possible phase shift detector. The incident laser beam is linearly polarized at 45° to the TE direction. Because of the waveguide resonance, the TE wave acquires a phase shift due to the presence of the atoms; the TM wave is affected only negligibly. If we ignore the difference in the TE and TM phase shifts upon total internal reflection, the analyzer would be a $\frac{1}{4}$ wave plate with its axes along the TE and TM directions, followed by a polarizing beam splitter whose axes were at 45° to the TE direction. Since we cannot ignore the TE and TM phase shifts on reflection, a real analyzer would require some other birefringent retarder in front of the polarizing beam splitter such that in the absence of the atoms, the outputs of the two ports are equal.

Another possibility is FM spectroscopy of the atoms in the evanescent wave [15]. The modulation frequency must be large enough that the carrier and the FM sidebands all get different phase shifts due to the bouncing atoms. Detecting the phase shifted carrier and sidebands and demodulating at the FM frequency is analogous to recording the interference between the TE and TM waves described above. This technique has the advantage of being at high frequencies, far from typical mechanical and laser noise sources. Our list of possible detection schemes is by no means exhaustive. Detailed experimental work must be done to evaluate various detection strategies, and we are beginning this study in our laboratory.

VI. CONCLUSION

The main point of this paper is to show that a laser phase shift measurement can detect bouncing atoms and to show how to choose the different experimental parameters to optimize the measurement. We conclude that with a bare prism, a realistic estimate of the atomic density and a choice of evanescent-mirror detuning that renders spontaneous emission unlikely, the measurement is possible but probably extremely difficult. The use of a resonant waveguide, however, seems to offer a significant advantage and leads us to claim that the experiment is feasible.

In our discussion, we have treated the atoms as a continuous cloud characterized by an index. In the limit of very low atomic density, our problem becomes that of the detection of

a single atom in an evanescent wave. The possibility of measuring the passage of a single atom obviously has important consequences for the coherence of the de Broglie wave describing the center-of-mass motion of the atom: if the mirror were in one path of an atom interferometer, and if one could tell where the atom was, the fringes would disappear. Therefore it is clear that there must be a mechanism in the evanescent-wave mirror that can destroy atomic coherence, even when spontaneous emission is avoided. The mechanism is related to the effect of intensity noise on the laser. Laser intensity noise causes the atomic mirror and therefore also the phase shift of the de Broglie wave to fluctuate. There is of course no fundamental lower limit on laser intensity fluctuations: squeezing techniques [16] can reduce laser intensity noise below the shot-noise limit. This reduction, however, is accompanied by an increase in the uncertainty of measurement of the reflected laser phase, and in our case these fluctuations would decrease our signal-to-noise ratio. In fact, our measurement may be considered a quantum nondemolition measurement of the passage of an atom in the evanescent wave; the quantity that is necessarily perturbed by the measurement is related to the phase of the de Broglie wave. This argument is made rigorous in Ref. [8].

Although when considering a single atom, the ability to detect its passage necessarily perturbs the coherent character of the reflection, this perturbation can be much reduced if one merely wishes to observe the passage of a cloud of atoms. In this case, the required sensitivity is much smaller, and so one can choose a combination of intensity and detuning that maintains the coherence of the reflection.

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